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# Hardness, Young's modulus and yield stress in mammalian mineralized tissues

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The hardness, Young's modulus of elasticity, tensile yield stress, ultimate stress and calcium content of 65 specimens from mammalian long bones and dental tissues were determined. The hardness was a very good predictor of the Young's modulus and yield stress, but a less good predictor of ultimate stress. The relationship in each case was nearly linear. All of these properties have the same type of non-linear relationship to the calcium content of the specimens. Over the range of calcium contents used here, hardness would be a useful guide to the mechanical properties of mineralized tissues in situations where conventional test specimens could not be produced, or where variations in mechanical properties over small distances are of interest.

## 1. Introduction

Biological hard materials are often difficult to test mechanically because they come in small pieces and awkward shapes. Although adequately large specimens can often be made of bone, some mammalian mineralized structures, such as the auditory ossicles or the components of teeth are too small to make standard specimens from. Furthermore, as our understanding of mineralized tissues deepens, it is becoming more important to be able to determine the variation in properties over small distances, considerably smaller than the size of standard test specimens. One possible method of estimating those mechanical properties of mineralized tissues that require large specimens would be to infer them from mechanical properties that can be tested on small specimens.

Microhardness is such a property. Microhardness is determined by measuring the size of the impression made by a diamond indentor, which is pressed into a surface with a small known load. Microhardness of bone and teeth has been investigated a number of times [1–5]. In most of these studies the hardness was considered as a mechanical property in its own right, not giving insight into other properties. However, Evans [2] reported a correlation coefficient of 0.52 between hardness and Young's modulus, and of 0.45 between hardness and tensile strength in human bone, without giving other information. Evans *et al.* [6] showed, among other things, that the Young's modulus could be inferred from a knowledge of hardness. Hodgskinson *et al.* [7] make some inferences about the Young's modulus of cancellous and neighbouring compact bone from microhardness measurements. Hardness is a somewhat unsatisfactory property because, unlike Young's modulus, tensile strength or fracture toughness, it is rarely important in its own right. However, there is a great deal of theoretical and experimental work showing that, if a material is capable of undergoing plastic flow, then hardness is related quite closely to yield stress [8].

The work of Evans *et al.* [6] was the first to show clearly the relationship between Young's modulus and hardness. However, it was based on only ten specimens, the Young's modulus was determined from a nearby piece of bone, not the bone on which the hardness determinations were made, and no measurements of yield stress could be made. In this paper, based on measurements on 65 specimens, it is shown that the microhardness of mammalian mineralized tissues is strongly correlated with yield stress, over a range of stresses from 30 to 180 MPa, and even more strongly correlated with Young's modulus, over a range of 5 to 30 GPa, and that it would be useful in estimating these mechanical properties. The relationship between hardness and ultimate tensile stress is clear, but less strong. We also show that, where comparisons can be made, the results of Evans *et al.* are completely consistent with these results.

## 2. Materials and methods

Specimens were machined from the compact bone of various mammals, and from the dentine and cementum of a Narwhal's tusk (Table I). These specimens were chosen, using previous knowledge, to give a good range of values of Young's modulus. About one-third of the specimens were from Polar bears, because we had available a range of ages which gave, as a result, a good range of values of Young's modulus. The specimens were loaded to failure in tension, wet, at a strain rate of $0.2\,\mathrm{sec}^{-1}$. Strain was measured with an extensometer on the central 11 mm gauge length. The Young's modulus of elasticity, yield stress and failure stress were determined from the stress–strain curve. Yield stress was taken to be the stress at which the curve deviated by a strain of 0.002 from the prolongation of the initial, linear part of the curve.

The hardness of the specimen was tested, wet, using a Leitz Wetzlar "miniload" microhardness tester, which produced square impressions whose diagonals were about 50 $\mu$m long. About six readings were taken

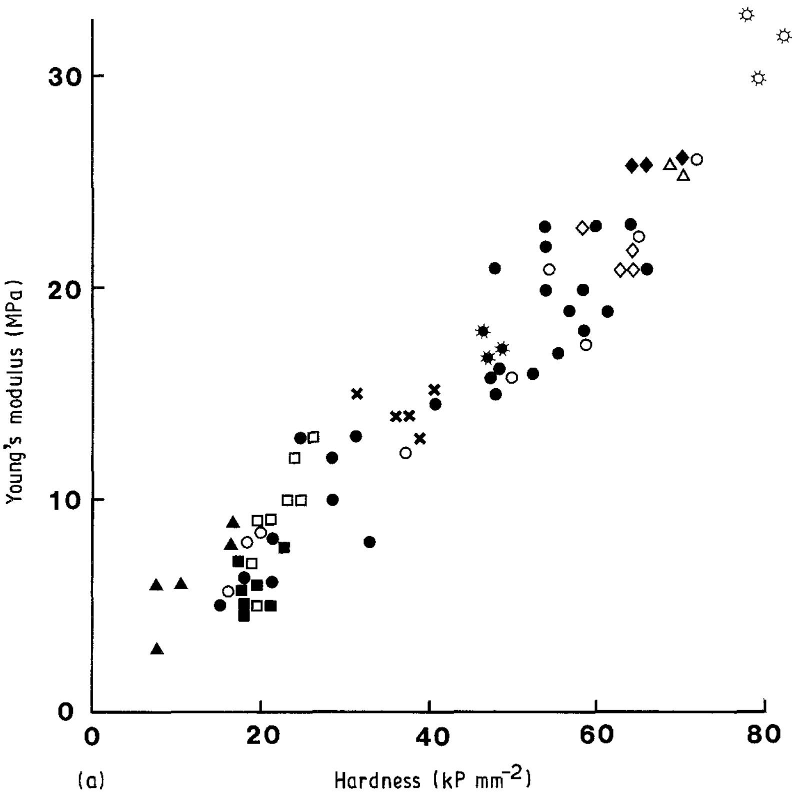


*Figure 1* Diagram of the interrelations of the various mechanical and chemical variables. The symbols in each diagram are the same. (▲) Red deer antler, (●) Polar bear femur (the different animals are not distinguished), (○) various specimens from [6] (■) Narwhal cementum, (□) Narwhal dentine, (×) Walrus humerus, (✱) Brown bear femur, (◇) Leopard femur, (◆) Cattle femur, (△) Horse femur, (☼) Axis deer femur. Hardness is traditionally measured in kilograms-force per square millimetre. 1 kP mm$^{-2}$ = 9.81 MPa.

from the machined and polished surface of that side of the specimen that lay tangential to the surface of the bone, antler or tusk. The hardness was determined from the shoulders of the specimen, which did not undergo yielding, a process which might possibly have affected the hardness.

The calcium content of the specimen was determined, colorimetrically, using the method of Sarkhar and Chauhan [9], from a small amount of material taken from just behind the fracture surface. Details of the mechanical testing are given in [10] and of the hardness testing in [7].

TABLE I The provenance of the specimens used in this study. All of the animals from which the specimens were taken were mature except the Polar bears, which included five animals, aged from 3 months to 7 years. Values for the five bears are shown separately (details in [11])

| Species | Specimen | *N* | Mean Young's modulus (GPa) | Mean yield stress (MPa) | Mean Vickers hardness | Calcium (mg g$^{-1}$) |
|---|---|---|---|---|---|---|
| Narwhal (*Monodon monoceros*) | Tusk dentine | 8 | 9.2 | 35.9 | 21.8 | 240 |
| Narwhal | Tusk cementum | 7 | 5.7 | 33.7 | 18.8 | 235 |
| Leopard (*Panthera pardus*) | Femur | 4 | 21.5 | 164.4 | 62.2 | 254 |
| Horse (*Equus caballus*) | Femur | 2 | 26.0 | 156.7 | 69.8 | 273 |
| Axis deer (*Axis axis*) | Tibia | 3 | 31.6 | 179.9 | 80.1 | 274 |
| Walrus (*Odobenus rosmarus*) | Humerus | 5 | 14.2 | 71.8 | 36.8 | 245 |
| Cattle (*Bos taurus*) | Femur | 3 | 25.9 | 141.9 | 67.0 | 267 |
| Brown bear (*Ursus arctos*) | Femur | 3 | 16.9 | 123.4 | 47.5 | 255 |
| Polar bear (*Ursus maritimus*) | Femur | 5 | 6.7 | 63.1 | 18.2 | 235 |
| | Femur | 5 | 11.2 | 83.0 | 29.4 | 251 |
| | Femur | 5 | 16.5 | 107.1 | 47.4 | 263 |
| | Femur | 5 | 18.6 | 123.3 | 58.7 | 259 |
| | Femur | 5 | 22.2 | 128.6 | 60.4 | 268 |
| Red deer (*Cervus elaphus*) | Antler | 5 | 6.1 | 44.3 | 11.7 | 196 |

*N*, sample size; Vickers hardness has the dimension of stress, one unit = 9.8 MPa; calcium, mean calcium content of dried, defatted bone.

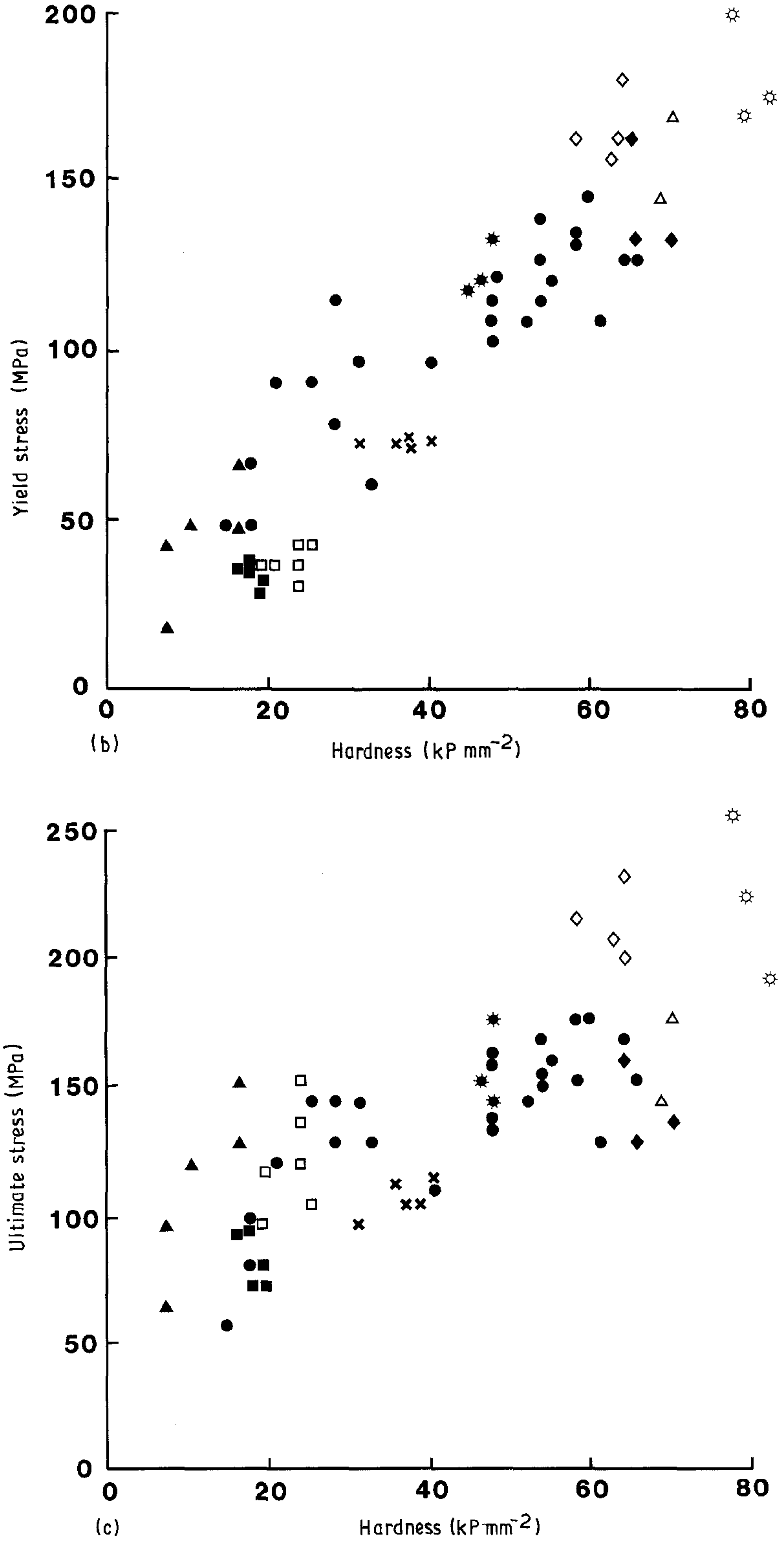


*Figure 1* Continued

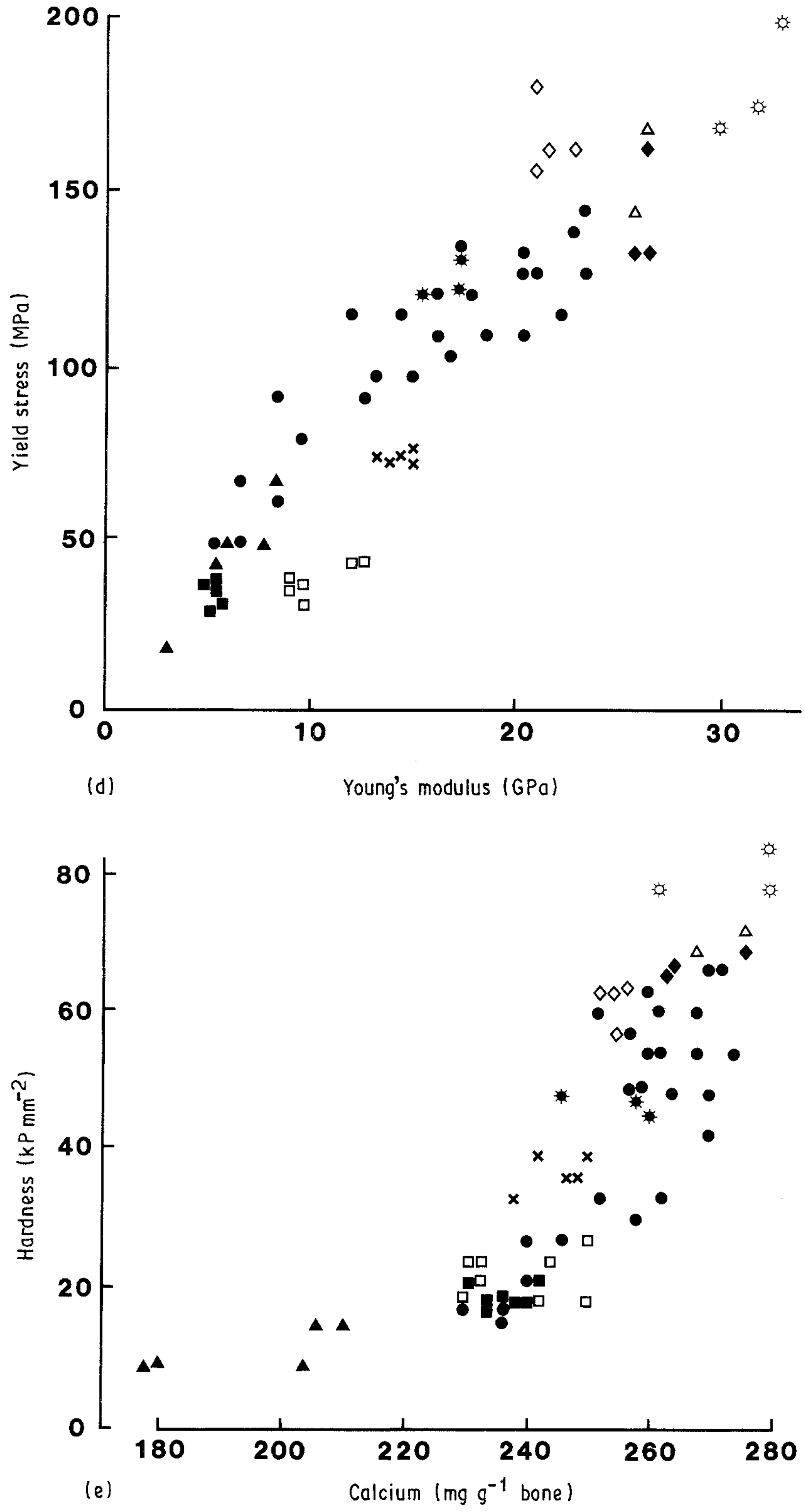
## 3. Results

The essence of the results is given in Figs 1a to g. Young's modulus is very highly correlated with hardness (Fig. 1a), and the relationship is essentially linear (a statistical analysis is given below). Although the relationship between yield stress and hardness is not quite as good (Fig. 1b), it is still very clear and essentially linear. There is a much looser relationship between ultimate tensile stress and hardness (Fig. 1c). Not surprisingly, since both Young's modulus and yield stress show a close, linear relationship with hardness, they are similarly related to each other (Fig. 1d).

Since hardness is supposedly determined mainly by yield stress [8], it is interesting that the relationship between hardness and Young's modulus is, in our data, closer than that between hardness and yield stress. It is possible that this effect may be partly artefactual, caused by the greater difficulty of measuring yield stress precisely.

Hardness will not, of course, actually determine the other mechanical properties; these mechanical relationships doubtless indicate a common relationship with another variable, almost certainly the mineral content of the tissues. Figs 1e, f and g show the relationships

*Figure 1* Continued



of hardness and the mechanical variables to calcium content. They are all very similar in their general form, which is strongly non-linear.

Table II gives the linear equations, derived from least-squares regression, relating Young's modulus and yield stress to hardness. The values of $R^2$ (93.3% and 85.2% for Young's modulus and yield stress, respectively) are high, suggesting that using the equations for predicting the mechanical variables from a knowledge of the hardness would be a reasonably reliable process. Furthermore, the constants in the equations (0.58 GPa for Young's modulus and 8.2 MPa for yield stress) are quite small, showing that the relationship is not only reasonably linear, but the variables are also nearly proportional to each other. However, it could be that a power law might fit the results more closely. Indeed, the data of Evans *et al.* [6] fit a square-root relationship more closely than they fit a linear equation. Transforming the variables to logarithms and performing a linear regresson is a straightforward way of testing for this. This produces equations of the form $y = ax^b$. Table II shows the results. The estimated exponents (0.92 and 0.88) are indeed less than unity. However, it can be seen that the values for $R^2$ for the equations on transformed data are less than those from the raw data. The reason for this is probably that the values near the origin become quite scattered when transformed (Figs 1h and i). The

*Figure 1* Continued

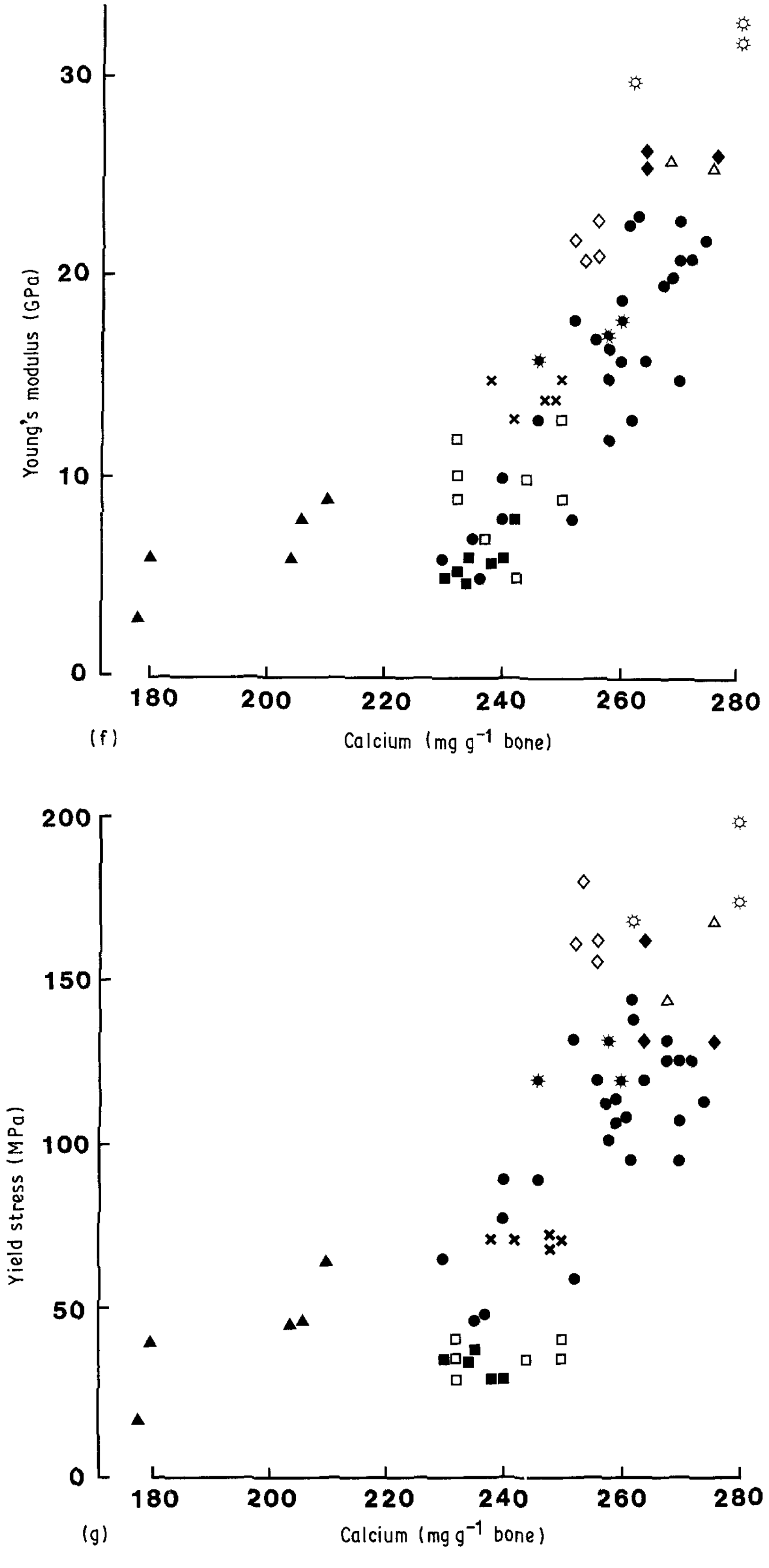


TABLE II Equations relating the mechanical variables to hardness. Equations 1a and b are derived from linear regression of untransformed data, Equations 2a and b from linear regression analysis of logged data, and Equations 3a and b from power-law analysis of untransformed data

Young's modulus = $0.58 + (0.36 \times \text{hardness})$, $R^2 = 93.3\%$ (1a)
Yield stress = $8.16 + (2.12 \times \text{hardness})$, $R^2 = 85.2\%$ (1b)

Young's modulus = $0.49 \times \text{hardness}^{0.92}$, $R^2 = 90.4\%$ (2a)
Yield stress = $3.53 \times \text{hardness}^{0.88}$, $R^2 = 78.3\%$ (2b)

Young's modulus = $2.56 + (0.142 \times \text{hardness}^{1.2})$, $R^2 = 93.6\%$ (3a)
Yield stress = $14.7 + (1.33 \times \text{hardness}^{1.1})$, $R^2 = 85.3\%$ (3b)

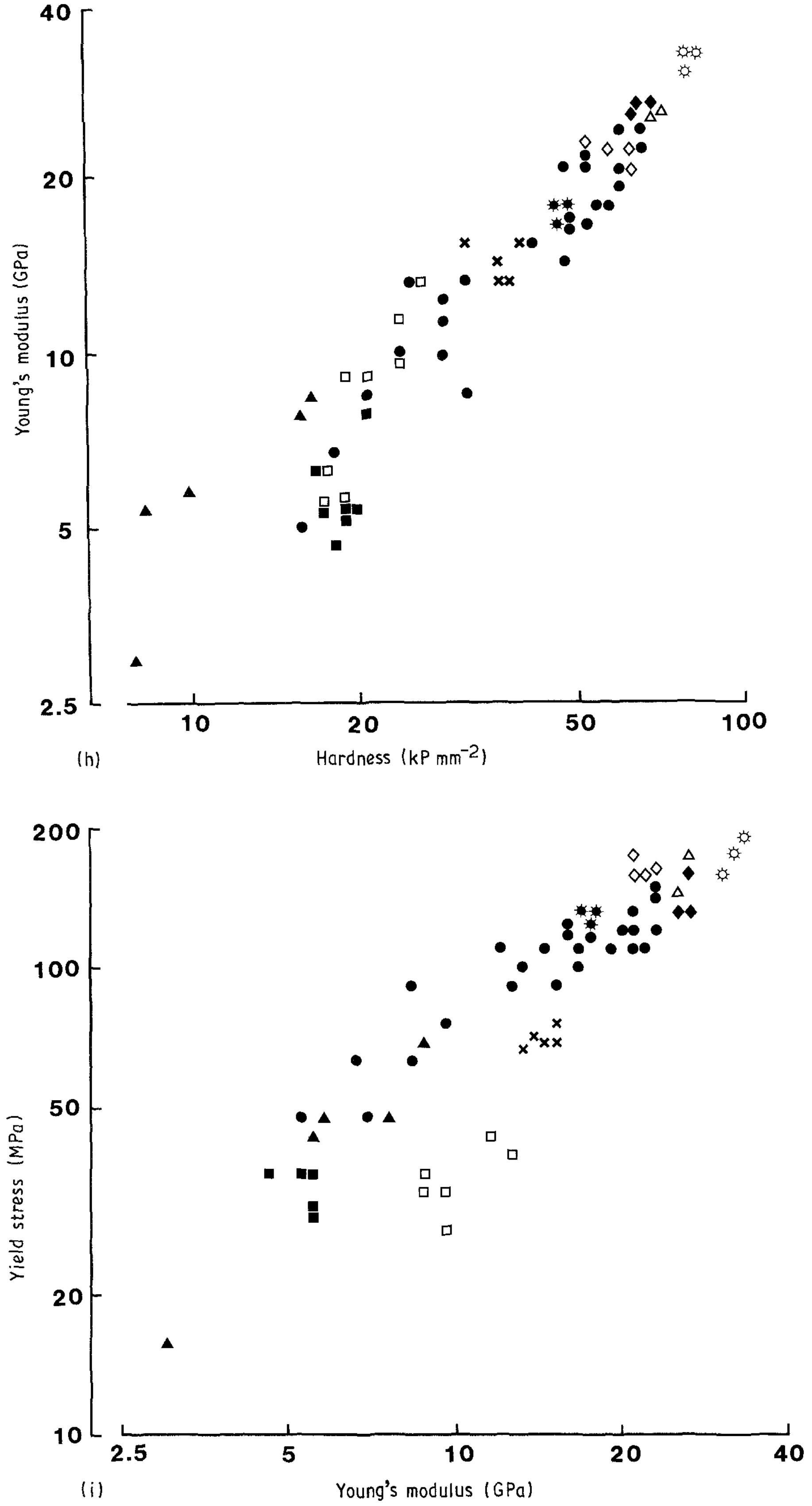


*Figure 1* Continued

fact that the data become rather heteroscedastic should make us wary of performing linear regressions on the transformed data.

Instead we have fitted power-law equations of the form $y = a + cx^b$. The results are also shown in Table II. The exponents (1.2 and 1.1 for Young's modulus and yield stress, respectively) are greater than unity, rather than being less than unity, as found in the transformed data. This is probably caused by the fact that equations of this form can have a non-zero intercept with the $y$-axis, which the equations of the transformed values do not. Certainly, although there is an increase in $R^2$ compared with the linear equation, it is extremely small, and the difference in the predicted values over the range where using these power-law equations and the linear equations as appropriate is also small. Furthermore, the equations with exponents greater than unity have larger values for the intercept, whereas common sense would suggest that, at zero hardness, both Young's modulus and yield stress should also be zero. All in all, it is probably reasonable to take the exponent of unity (Equations 1 and 2) as being a good model of the real situation.

## 4. Discussion

These results show that microhardness is a good predictor of both Young's modulus and yield stress, but not ultimate stress, over the range of mineralizations tested here. The relationships are close to linear, the best estimates of the exponents being 1.2 and 1.1, respectively. It is reassuring that the data of Evans *et al.* for Young's modulus as a function of hardness agree well with our results (Fig. 1a) despite the fact that hardness was tested on a different machine, and that their specimens included mammals, birds and reptiles, compared with our more restricted data set of mammals only. Our data set includes dental tissues, and they also fit into the general pattern quite well, although the cementum values for Young's modulus are somewhat low for a given hardness (Table I).

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## References

1. D. CARLSTRÖM, *Experientia* **10** (1954) 171.
2. F. G. EVANS, "Mechanical Properties of Bone" (Charles C. Thomas, Springfield, Illinois, 1973).
3. G. LENART, I. TOTH and J. PINTER, *Acta Biochim. Biophys. Acad. Sci. Hung.* **3** (1968) 205.
4. A. PIZZOFERRATO *et al.*, in "Ceramics in Surgery", edited by P. Vincenzini (Elsevier, Amsterdam, 1983) p. 149.
5. M. D. WEAVER, *J. Bone Joint Surg.* **48A** (1966) 273.
6. G. P. EVANS *et al.*, *J. Mater. Sci.: Mater. Med.* **1** (1990) 38.
7. R. HODGSKINSON, J. D. CURREY and G. P. EVANS, *J. Orthop, Res.* **7** (1989) 754.
8. A. KELLY and N. H. MACMILLAN, "Strong Solids" (Clarendon Press, Oxford, 1986).
9. B. C. SARKHAR and U. P. S. CHAUHAN, *Analyt. Biochem.* **20** (1967) 155.
10. J. D. CURREY, *J. Biomech.* **21** (1987) 131.
11. J. D. CURREY and C. POND, *J. Zool., Lond.* in press.

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